Chapter - 4 20/10/21

PARTIAL order Relation

Introduction :- (Defination)

-> Any relation are on set (S) is called partial order relation if for any 3 arbitrary element a, b, c ∈ S

relation have three types :-

1. R is reflexive

2. anti symmetric

3. R is transitive

α Reflexive relation :- aRa, or $a \leq a$

α Anti symmetric relation :- aRb & bRa $a = b$ the or $a \leq b$ or $b \leq a$ then $a = b$

α transitive relation :- aRb & $bRc \Rightarrow aRc$
$a \leq b, b \leq c \Rightarrow a \leq c$

[∵ $a \cdot 0' = 0$ is new law]

Theorem:- The order relation less than equal to is partial order relation in Boolean algebra.

Proof of one:- reflexive relation.

For each $a \in B$, we have $a \cdot a' = 0$, this implies that $a \leq a$

Hence the relation $\leq$ reflexive

(ii) Anti-symmetric relation:- Let $a, b \in B$ and $a \leq b$, $b \leq a$

and we known that:-

a, $a, b \in B$

$$a \leq b = a \cdot b' = 0 \quad \text{—eq(i)}$$

$$b \leq a = b \cdot a' = 0 \quad \text{—eq(ii)}$$

$$a = a \cdot 1 \quad \text{—(iii)} \quad [\text{identity law}]$$

$$= a \cdot (b + b') \quad [\text{By complementary law}]$$

$$= a \cdot b + a \cdot b' \quad [\text{By distributive law}]$$

$$= a \cdot b + 0 \quad [\text{By using eq (i)}]$$

$$= a \cdot b$$

Again

$$b = b \cdot 1 \quad [1 \text{ is multiplicative identity}]$$

$$b = b \cdot (a + a') \quad [\because a + a' = 1]$$

$$= b \cdot a + b \cdot a' \quad [\text{By distributive law}]$$

$$= b \cdot a + 0 \quad [\text{By using eq(2)}]$$

$$= b \cdot a \quad [\text{By commutativity}]$$

$$= a \cdot b$$

(iii) Transitive relation:- Let $a, b, c \in B$ and $a \leq b$, $b \leq c$

$$a \leq b = a \cdot b' = 0 \quad \text{--- (1)}$$

$$b \leq c = b \cdot c' = 0 \quad \text{--- (ii)}$$

Now

$$a \cdot c' = (a \cdot 1) \cdot c' \quad [\text{identity law}] \quad [\because a = a \cdot 1]$$

$$= a \cdot (b + b') \cdot c' \quad [\because 1 = b + b']$$

$$= a \cdot b + a \cdot b' \cdot c' \quad [\text{By distributive law}]$$

$$= a \cdot b + 0 \cdot c' \quad [\text{By use of 1}]$$

$$= (a \cdot b) \cdot c' \quad [\because \text{By complementary law}]$$

$$= a \cdot (b \cdot c') \quad [\text{By associativity law}]$$

$$= a \cdot (0) \quad [\text{By use eq (ii)}]$$

$$= 0 \quad [\because a \cdot 0 = 0]$$

Thus $a \leq b$, $b \leq c$, $\Rightarrow a \cdot c' = 0$

$$\Rightarrow a \leq c$$

Hence the relation $\leq$ is transitive.

Hence the order relation $\leq$ is partial order relation.

Theorem 3 :- (Homework) [Denoated By lub sup(A)]

☆ LEAST Upper Bound :-

Let $(B,+,\cdot,')$ be a Boolean algebra. Let A be a subset of B. An element $b \in B$ is called an upper bound of A if and only if $a \le b \ \forall\ a \in A$

conditions :-

(a) c is an upper bound of the set A and

(b) $c \le b$ for every upper bound b of A

☆ Greatest lower Bound :- [denoated By glb(A)]

Let $(B,+,\cdot,')$ be a Boolean Algebra let A be a subset of B An element $b \in B$ is called a lower bound of A if only if $b \le a \ \forall\ a \in A$.

conditions :-

(a) c is an lower bond of the set A, and.

(b) $b \le c$ for to every lower bond b of A

Theorem :-

To prove that for any two elements a, b of a Boolean Algebra:

(i) $a+b$ = least upper bound of a and b
$a+b = \sup\{a,b\}$

(ii) $a \cdot b$ = greatest lower bounded of a & b
$a \cdot b = glb\{a,b\}$

(i)
Proof :- firstly we shall prove that $a+b$ is an upper bound of this set $\{a,b\}$.

So this will shall prove that
$a \le a+b \quad b \le a+b \quad \left[\because\ b=(a+b),\ a\cdot b'=0,\ b\cdot a'=0\right]$

i.e, $a\cdot(a+b)'=0$
$a\cdot(a'\cdot b')=0$
$(a\cdot a')\cdot b'$ [By Associativty law]
$0\cdot b'$ $[\because\ a\cdot a'=0]$ $[\because\ 0\cdot b'=0]$
0

Hence L.H.S prove R.H.S [$\because$ L.H.S = R.H.S]

So, $a\cdot(a+b)'=0$ or $a \le (a+b)$

i.e $b\cdot(a+b)'=0$

$b\cdot(a'\cdot b')=0$ [$\because$ By complementary law]

$a'\cdot(b\cdot b')$ [$\because$ By Associativity law]

$a'\cdot 0$ [$\because b\cdot b'=0$] [$\because a'\cdot 0=0$]

0

Hence ~~we~~ prove

L.H.S = R.H.S

So, $b\cdot(a+b)'=0 = b\leq(a+b)$

Thus from relation (1) and (2) it follow that $(a+b)$ is an upper bound of element a and b.

Now we shall prove that $a+b$ is lub of a and b. and suppose that c is also an lub of a and b. so

$a\leq c,\ b\leq c$

$a\cdot c'=0 \quad b\cdot c'=0$

$(a+b)\cdot c'=0$

$(a+b)\cdot c'$

$c'\cdot(a+b)$ [By commutativ law]

$(c'\cdot a)+(c'\cdot b)$ [By Distributive law]

$= a\cdot c' + b\cdot c'$ [By commutativity law]

$= 0+0$

$= 0$

~~b.c'=0 that~~ $(a+b)\cdot c'=0 \Rightarrow a+b\leq c$

$\text{lub}\{a,b\}=a+b.$

Ex :- 3 If B is Boolean Algebra :-

If $x\cdot y\geq z \Leftrightarrow x\geq z$ and $y\geq z$

Soln :- $x\cdot y\geq z \Leftrightarrow z\leq x\cdot y$

$x\geq z,\ y\geq z \Leftrightarrow x\geq z,\ \&\ y\geq z$

we prove that

$z\leq x\cdot y \Leftrightarrow z\leq x\ \&\ z\leq y$

let

$z\leq x\cdot y$ then

$z\leq x\cdot y,\ x\cdot y=\text{glb}\{x,y\}$

$z\leq x\cdot y,\ x\cdot y\leq x$ and $x\cdot y\leq y$

$z\leq x,\ z\leq y.$ ....(1)

For conversely:-

Let $z \le x$ and $z \le y$ then

$z \le x \Rightarrow z + x = x$ -(2) [According to theorem No (3)]

$z \le y \Rightarrow z + y = y$ ...(3) [$\because a + b = b$]

Now, $z + x \cdot y$

[For distributive law]

$z + x \cdot y = (z + x) \cdot (z + y)$

From eq (2) and (3)

$$z + x \cdot y = x \cdot y$$

$$z \le x \cdot y \quad \text{----(4)}$$

Hence from eq (1) and eq (4) we have

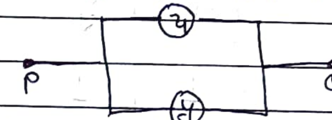
$$z \le x \cdot y \Leftrightarrow z \le x \;\&\; z \le y$$

$$x \cdot y \ge z \Leftrightarrow x \ge z \;\&\; y \ge z$$

Algebra of electric circuits:-

In Parallel:-

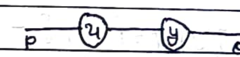
when two switches x and y are connected in parallel, then symbolically they are represented by the nation $x + y$.



$x + y \Leftrightarrow$ two switches x and y are connected in parallel.

In Series:-

when two switches x and y are connected in series, then symbolically they are represented by the nation $x \cdot y$.



$x \cdot y$ or $xy \Leftrightarrow$ two switches x and y are connecte in series.

Truth Table:- (In parallel) ~~switching~~

when switch is x and y are connected in parallel

$\Rightarrow$

x	y	x + y
1	1	~~0~~1
1	0	1
0	1	1
0	0	0

relation between close and open switches:- If x represent the close switches the x' will represent an open switch conversely if x represent an open switch the x' will be (a closed switch)

Truth Table:- (In series)

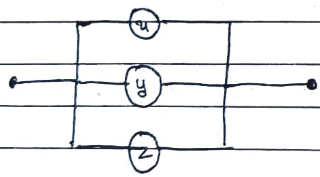
when switches x and y are connected is series

x	y	$x \cdot y$
1	1	1
1	0	0
0	1	0
0	0	0

☆ Switching circuit:-

Switching function:-

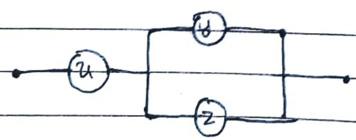
Switching circuit — Switching function $f(x, y, z)$

(i) 

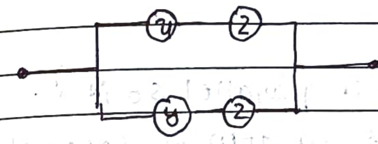
$x+y+z$

(ii) 

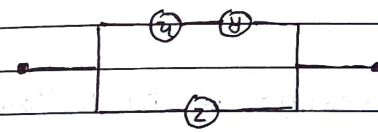
$x \cdot y \cdot z$

(iii) 

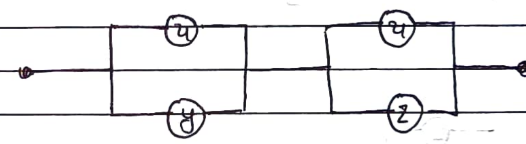
$x \cdot (y+z)$

(iv) 

$x \cdot z + y \cdot z$

(v) 

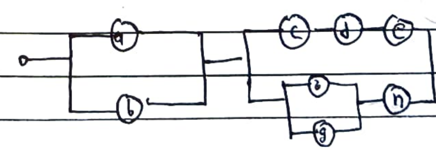
$x \cdot y + z$

(vi) 

$x+y \cdot x+z$

Ex:-1 find the switching netlory switching function or polynomial

as:



In word form:-

1. Switch a and b are connected in parallel. It is represent by $(a+b)$

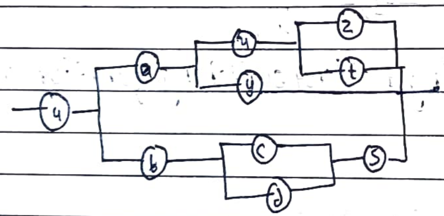
2. Switches c and d, e are connected in series which is represented by (c·d·e)

3. Switches z and g are connecting in parallel so it is represented by (z+g), This system is connected in series with switch 'h' now it is represented by (z+g)·h

Hence the given circuit is represent by the following polynomial.

$$[(a+b)\cdot(c\cdot d\cdot e)+(z+g)\cdot h]$$

Q 7.



In word form:-

1. Switch a and b are conneted in parallel ~~that~~ It is represented by (a+b)

3. Switch a is connected in series with x and y the switch x and y are connected in parallel It is represented by a·(x+y)

4. switch x is connected in series with z and t and switch z and t are connected in || It is represented by x·(z+t)

5. ~~b~~ switch b is connected in series ~~with~~ with c and d and switch c and d are connected in || c and d are connected in s. b·(c+d)·s

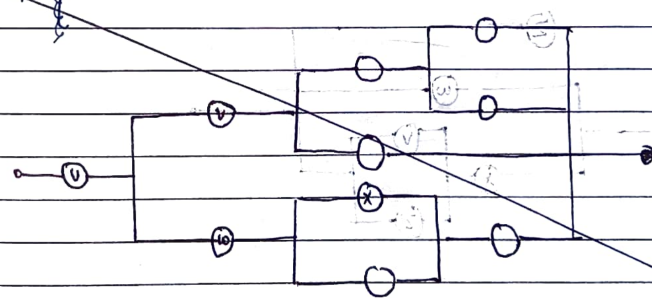
6. directly all switches are series connect in u.

$$\cancel{u\cdot[v\cdot(x\cdot(z+t)+y \cdots (c+d)\cdot s]}$$

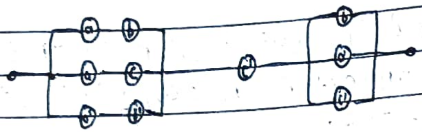
$$u\cdot[v\cdot(x\cdot(z+t)+y+w[a(x+y)\cdot s]$$

Homework:-

7.



Homework:-

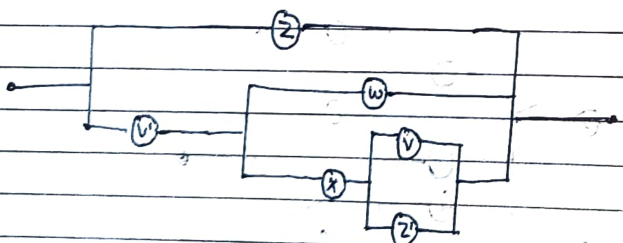
6. 

Soln:- (1) The first circuit clearly represent by the polynomial $(ab + ac + a'b')$!

(2) The second circuit consists of ~~which~~ switch c' only
The third circuit is clearly represented by the polynomial $(b + a' + c')$.

(3) Since above three circuit are connected in series hence the whole given circuit (6) is represented by the following polynomial:

$$[(ab + a\cdot c + a'b')\cdot c' \cdot (b + a' + c')].$$

8. 

Soln:-

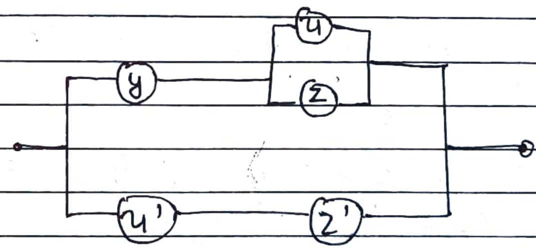
(i) v and z' are in parallel, which is represend by $v+z'$

(ii) u and $(v+z')$ are in series, which is represented by $u(v+z')$.

(iii) w' and $u(v+z')$ are in parallel which is represented by $\{w' + u(v+z')\}$.

(iv) v' and $\{w' + u(v+z')\}$ are in series which is represented by $v'\{w' + u(v+z')\}$ and this represents the whole of the lower net.

$$[z + v'\{w' + u(v+z')\}].$$

4. 

Soln:- Switches u and z are connected in parallel, which is denoated $u+z$.

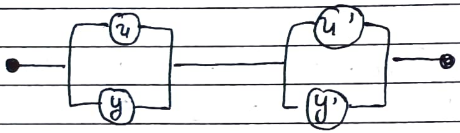
(ii) y and $(u+z)$ are connected is series which is denoated by $y(u+z)$.

(iii) x' and z' are connected in series, which is denoted by $x'z'$.

(iv) Now $y(x+z)$ and $x'z'$ are connected in parallel, which is denoted by $y(x+z)+x'z'$

$$[y(x+z)+x'z'.]$$

3.



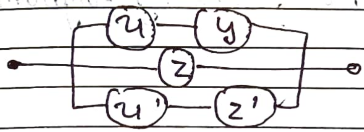
Soln:- (i) Switches x and y are connected in parallel, which is denoated by $x+y$.

(ii) switches x' and y' are connected in parallel which is denoated by $x'+y'$

(iii) Now $(x+y)$ and $(x'+y')$ are connected in series which represented by $(x+y)\cdot(x'+y')$.

$$[(x+y)\cdot(x'+y').]$$

2.



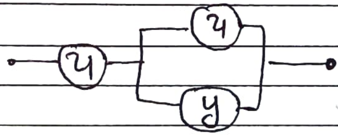
Soln:- (i) Circuit x and y are connected in series, which is denoted by xy.

(ii) Switches x' and y' are connected in serie which is denoted by $x'y'$

(iii) Now xy, z and $x'y'$ are connected in parallel, which is represent by $(xy+z+x'y')$.

$$[xy+z+x'y']$$

1.



Soln:- (i) Switches x and y are connected in parallel and which is represented by $x+y$.

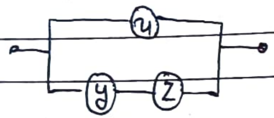
(ii) This system is connected in series with switch x, which is represented by $x\cdot(x+y)$

$$[x\cdot(x+y)]$$

Ex:-2 Design the circuits for the following polynomials:

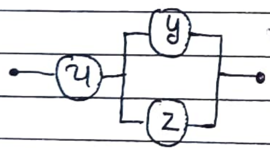
(i) $x+yz$,

Soln:- Switches x and yz are in parallel
Switches y and z are in series
Hence diagram of the given circuit.



(ii) $x(y+z)$,

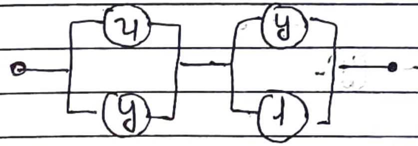
Soln:- (i) Switche x and $(y+z)$ are in series while y and z are in parallel.
Hence diagram of the given circuit



(iii) $(x+y)(z+1)$,

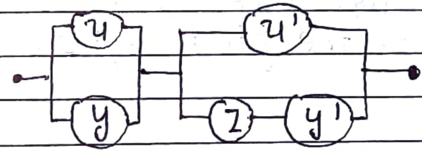
Soln:- the given net consists of two circuits in series
(ii) first circuit is $x+y$, in which x and y are in parallel.
(iii) second circuit is $z+1$ in which z and 1 are in parallel

Hence diagram of the given circuit



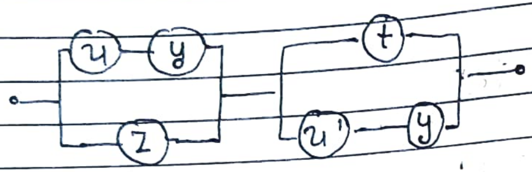
(iv) $(x+y)(x'+zy')$

Soln:- (i) Switche x and y are connected in parallel
(ii) Switche x' and zy' are connected in parallel
in while z and y' are connected in series.
(iii) whole switche are connected in series.
Hence diagram of the given circuit.



(v) $(xy+z)(t+x'y)$.

Soln:- (i) switches xy and z are connected in parallel
in while x and y are connected in series
(ii) Switche t and $x'y$ are connected in parallel
and while x' and y are in series
(iii) whole circuit are in series
Hence diagram in the given below:-



(vi) $xyz + xy(sz + tu)$

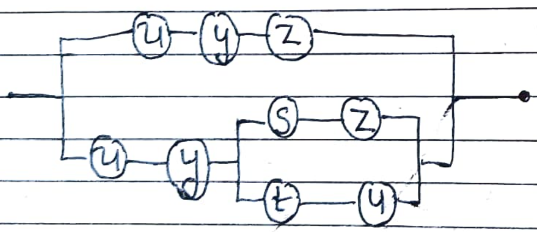
Soln:- Here switches x and y or z are connected in series.

(ii) switches ~~or~~ xy are series connected in $(sz+tu)$ and s and z are connected in series ~~on~~ while t and u are connected in series

(iii) s and z are parallel connected on tu

(iv) whole circuit are connected in parallel.

Hence diagram of the given series.



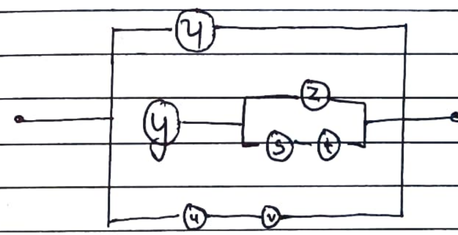
(vii) $x + y(z + st) + uv$.

Soln:- (i) switche x and y are connected in parallel and $(x+y)$ are series connected in $(z+st)$

(ii) z are parallely connected in st and s/t are ~~are~~ connected is series.

(iii) u and v are connected in series and $(z+st)$ are parallely connected on uv.

Hence diagram of the given circuit:-



(viii) $x[y(z+w) + z(u+v)]$.

Soln:- (i) first circuit is of x only.

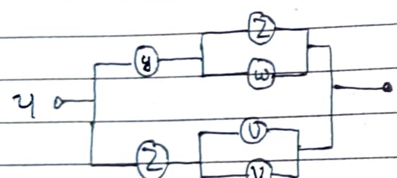
(ii) second ~~s~~ circuit is $y(z+w)$ in y and $(z+w)$ are connected in series while z and w are connected in parallel.

Homework :- all example :-

(iii) circuit z(u+v) in which z and (u+v) are connected in series while u and v are connected in parallel.

(iv) circuit y(x+w) and z(u+v) are in parallel and x is connected in series.

Hence diagram of the given below :-

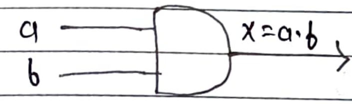


Ex :- 3

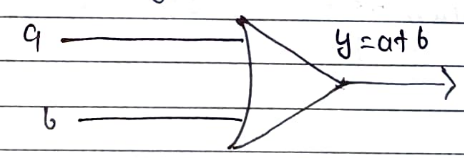
Logic Gates and Circuits :-

(i) AND-element or AND-gate :-

If there are two or more input then the output through their AND-gate (or AND-element) is a function of inputs and this function is obtained by the product of inputs.

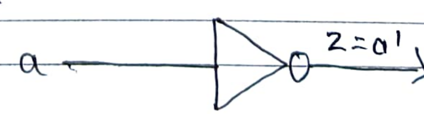


(ii) OR-element or OR-gate :- If there are two or more input then the output through their or-gate is a function of inputs and this function is obtained by the Additions of inputs.



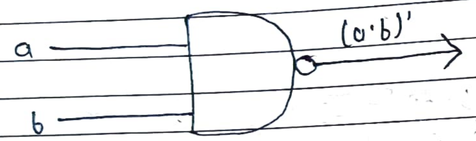
(iii) NOT-element or Inverter :-

Not element changes the input to the complement of it i.e. the output through the NOT-element is the complement of input.

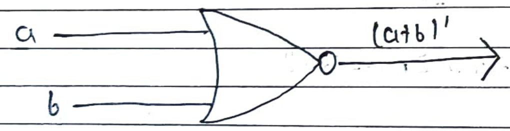


Combination of gates:-

1. ~~AND~~ NAND gate:- It is equivalent to ~~AD~~ AND gate followed by NOT gate if A and B are two inputs there ~~a in~~ output is NAND gate is $(a.b)'$



2. NOR-Gate:- It is equivalent to an OR-Gate followed by a NOT-Gate if a and b are two inputs then ~~teit~~ of output thuyout NOR-gate is $(a+b)'$



Truth Table of NAND Gates:-

a	b	(a.b)	(a.b)'
1	1	1	0
1	0	0	1
0	1	0	1
0	0	0	1

Truth Table of NOR Gates:-

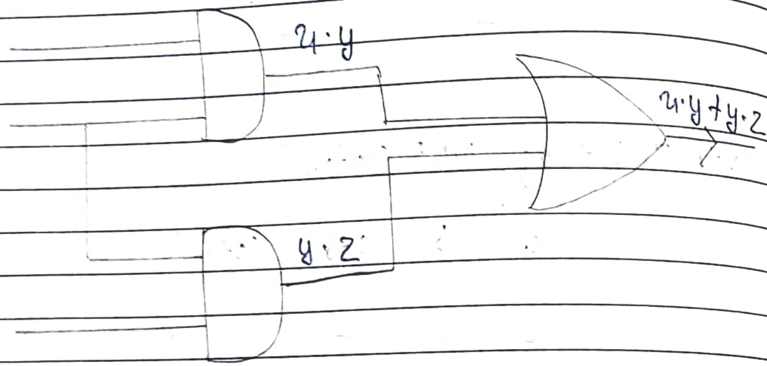
a	b	(a+b)	(a+b)'
1	1	1	0
1	0	1	0
0	1	1	0
0	0	0	1

Ho

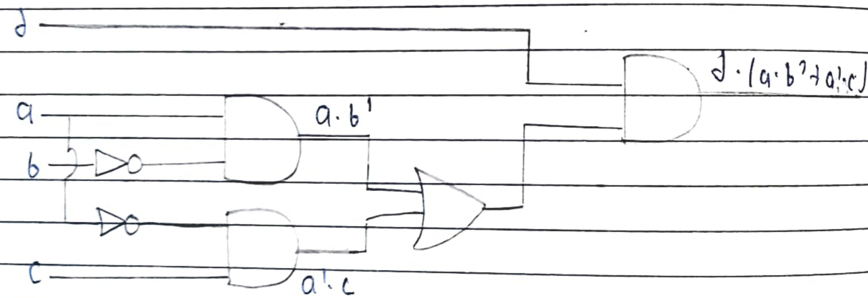
Ex:-1 Draw the logic circuit for each of the following Boolean expressions.

(i) $x \cdot y + y \cdot z$

Soln:-

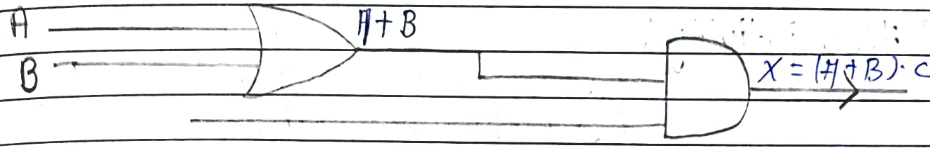


(ii) $d \cdot (a \cdot b' + a' \cdot c)$



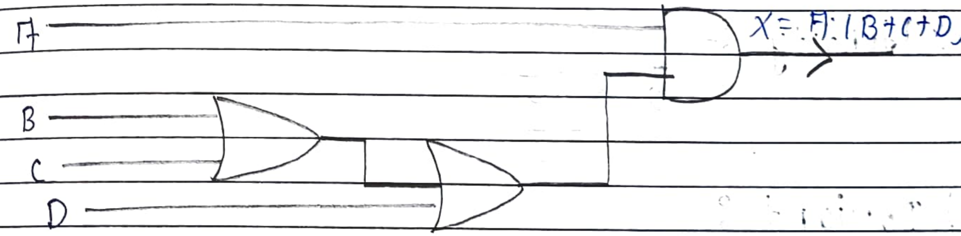
(iii) $y = (A+B) \cdot C$

Soln:-



(iv) $X = A(B+C+D)$

Soln:-



Homework:- Theorem:-

The set ~~&~~ Fn of all switching functions of n variables forms a Boolean algebra with respect to operations '+', '·', and '''. In other words the following postulates ~~hod~~ hold for all $f, g, h \in F_n$'

(i) Commutativity :-

$$f+g = g+f$$
$$f\cdot g = g\cdot f$$

(ii) Associativity :-

$$(f+g)+h = f+(g+h)$$
$$(f\cdot g)\cdot h = f\cdot(g\cdot h).$$

(iii) Distributive :-

$$f(g+h) = fg+fh$$
$$f+gh = (f+g)(f+h)$$

(iv) Identity :-

$$f+0 = f.$$
$$f\cdot 1 = f.$$

(v) Dominant :-

$$f+1 = 1$$
$$f\cdot 0 = 0$$

(vi) Idempotent :-

$$f+f = f$$
$$f\cdot f = f.$$

(vii) Complementation :-

$$f+f' = 1$$
$$f\cdot f' = 0$$

(viii) De-Morgan's law :-

$$(f+g)' = f'\cdot g'$$
$$(f\cdot g)' = f'+g'$$

(ix) Involution :-

$$(f')' = f$$

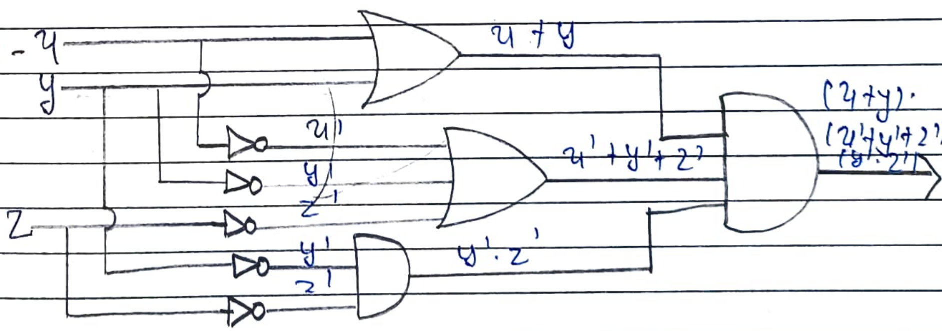
(x) Absorption :-

$$f+fg = f$$
$$f\cdot(f+g) = f$$

Ex:-2 Draw the logic circuit for each of the following expression:-

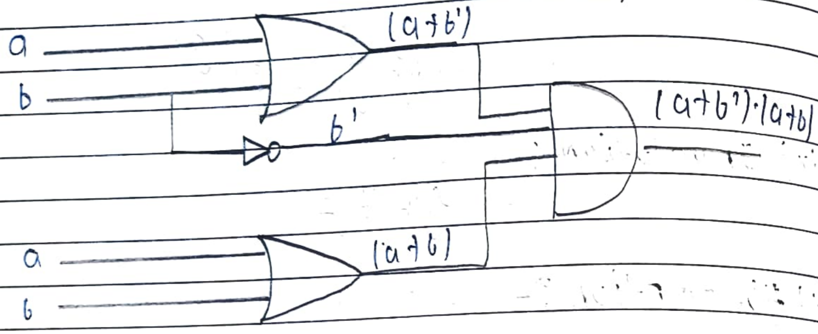
(i) $(x+y)\cdot(x'+y'+z')\cdot(y'\cdot z')$

Soln :-



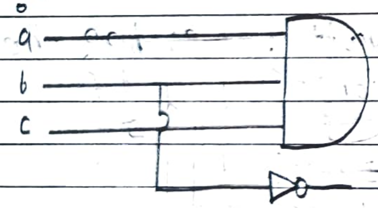
(ii) $(a+b')\cdot(a+b)$

Soln:-



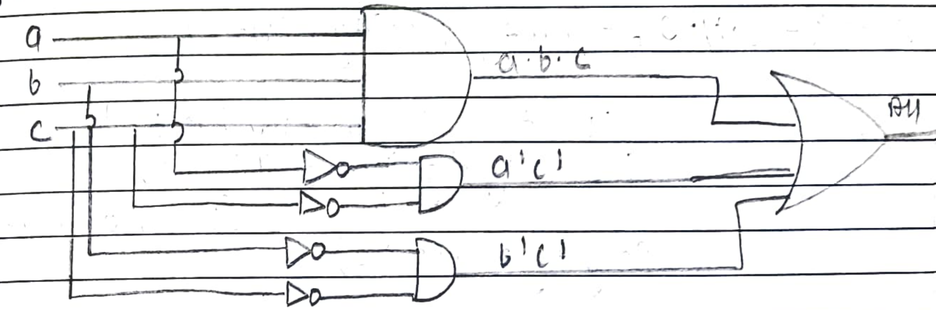
Ex:-3
~~(iii)~~ $y = ab'c + abc' + ab'c'$

Soln:-



Ex:-4 $x = abc + a'c' + b'c'$

Soln:-



Prove the following Boolean identities:-

(i) $(x'y+z)'(x'+z)' + (x'y+z)(x'+z) = x+y+z$

Soln:- L.H.S:- $((x'y+z)'+(x'+z))\cdot((x'+z)'+(x'y+z))$

$((x'y)'z'+x'+z)(xz'+x'y+z)$

$((x+y')z'+x'+z)(xz'+x'y+z)$

$(xz'+z)+(y'z'+x)(x'y)$

$xz'+z+(y'z')x'y+xx'y$

$xz'+x'y+z$

$(z+xz'+x')(z+xz'+y)$

$(z+x'+x)(z+x'+z')(z+x+y)(z+z'+y)$

$(z+1)(1+x')(x+y+z)(1+y)$

$1\cdot 1(x+y+z)\cdot 1$

$x+y+z$.

(ii) $(x+y')(x'+y)(x+z) = \cancel{x+y+z}\,(x+y')(x'+y)(y+z)$

Soln:- L.H.S:- $(x+y')(x'+y)(x+z)$

$(xx'+xy+y'x'+yy')(x+z)$

$(0+xy+y'x'+0)(x+z)$ [By complementation law]

$[xy+y'x'](x+z)$

$xxy+y'x'x+zxy+zy'x'$

$xy+xyz+y'x'z$ $\quad [\because xx'=0$

$xyy+xyz+y'x'y+y'x'z$

$xy(y+z)+x'y'(y+z)$

$(xy+x'y')(y+z)$

$(x+y')(x'+y)(y+z)$

HOMEWORK:-

Ex:-3 For the following switching functions, draw a simpler switching circuit:

(i) $f(x,y) = x+x\cdot y$

Soln:- $f(x,y) = x+x\cdot y$

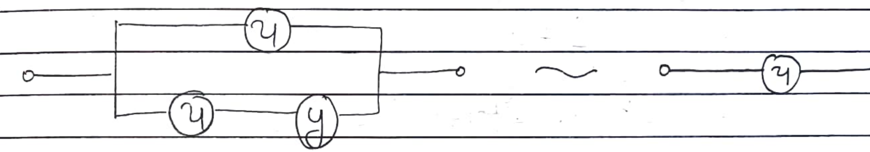
$= x\cdot 1+x\cdot y$

$= x\cdot(1+y)$

$= x\cdot(y+1)$ $\quad [(y+1)=1]$

$= x\cdot 1$

$= x$



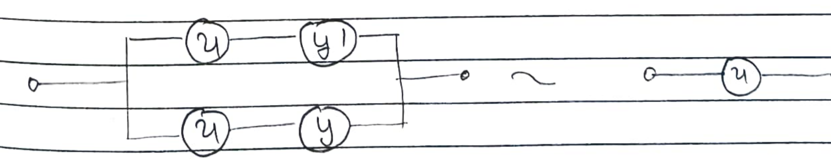
(ii) $f(x,y) = x\cdot y'+x\cdot y$

Soln:- $x\cdot y'+x\cdot y$

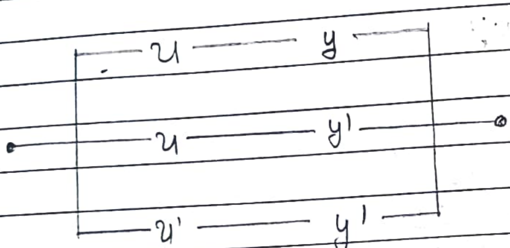
$x(y'\cdot y)$

$x\cdot 1$ $\quad [y'\cdot 1=1]$

x



Ex4:- Draw a simpler circuit for the following diagram and verify the equivalent circuit by truth table.



Soln:- $f(x,y) = (x\cdot y)+(x\cdot y')+(x'\cdot y')$

$= x\cdot(y+y')+(x'\cdot y')$

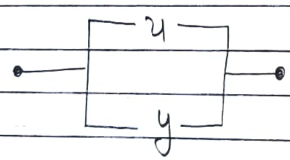
$= x\cdot 1 + x'\cdot y'$ $[\because x\cdot 1 = x]$

$= x + x'\cdot y'$

$= (x+x')\cdot(x+y)$ $[\because x+x' = 1$

$= 1\cdot(x+y)$

$= x+y'$



Verification by truth tables:-

x	y	$x\cdot y$	y'	$x\cdot y'$	x'	$x'\cdot y'$	$x\cdot y+x\cdot y'+x'\cdot y'$
1	1	1	0	0	0	0	1
1	0	0	1	1	0	0	1
0	1	0	0	0	1	0	0
0	0	0	1	0	1	1	1

x	y	y'	$x+y'$
1	1	0	1
1	0	1	1
0	1	0	0
0	0	1	1

Ex:- 6 Replace the switching function by a simpler switching circuit.

$f(x,y,z) = x\cdot y\cdot z + x\cdot y'\cdot z + x'\cdot y'\cdot z$

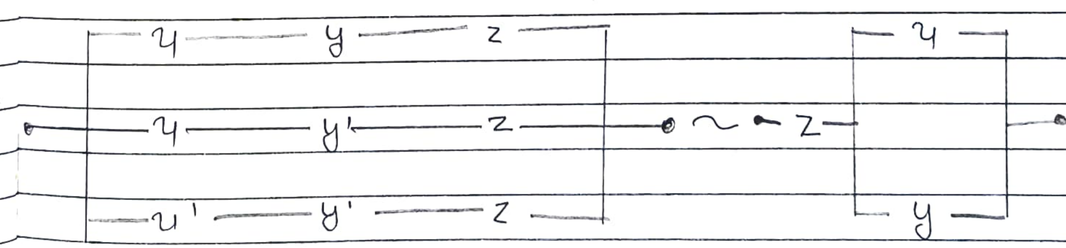
Soln:- $f(x,y,z) = x\cdot y\cdot z + x\cdot y'\cdot z + x'\cdot y'\cdot z$

$= (x\cdot y + x\cdot y' + x'\cdot y')\cdot z = z\cdot[x\cdot(y+y')+x'\cdot y']$

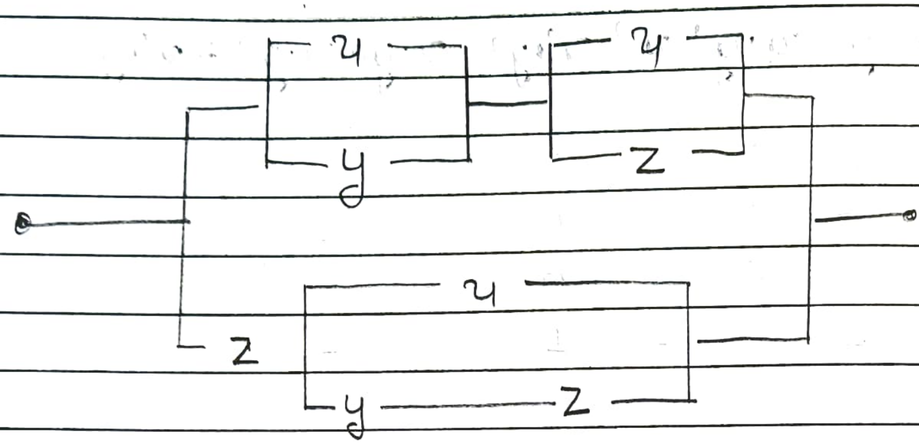
$= z\cdot[x\cdot 1 + x'\cdot y'] = z\cdot[x+x'\cdot y']$

$= z\cdot(x+x')\cdot(x+y')] = z\cdot(1\cdot(x+y')] = z\cdot(x+y')$

$= z\cdot(x+y')$



Ex:-7 Replace the following switching circuit by a simpler one



Soln:- $f(x,y,z) = (x+y)\cdot(x+z) + z\cdot(x+y\cdot z)$

$= x+y\cdot z + z\cdot(x+y\cdot z)$

$= (x+y\cdot z)\cdot 1 + z\cdot(x+y\cdot z)$

$(x+y\cdot z)\cdot(1+z)$

$(x+y\cdot z)\cdot 1$

$x+y\cdot z$